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322452(14)

B. E. (Fourth Semester) Examination,
April-May 2021

(New Scheme)

(CSE Branch)

DISCRETE STRUCTURES

Time Allowed : Three hours

Maximum Marks : 80

Minimum Pass Marks : 28

Note : *In each question Part (a) is compulsory and carries 2 marks. Solve any **two** part from (b), (c) and (d) each carry 7 marks.*

Unit-I

1. (a) State and explain Boolean algebra.

(b) Write the following functions into disjunctive normal form of 3 variables x, y, z

(i) $x' + y'$

(ii) $(x+y)\,(x'+z')$

(c) Prove that the structure $(B, +, \cdot, ')$ is a Boolean algebra where $B = \{ 0, 1 \}$ and $+$ and $\cdot$ are two binary operation and (') a unary operation of B, defined by the following tables

+	0	1
0	0	1
1	1	1

Table 1

·	0	1
0	0	0
1	0	1

Table 2

a	a'
0	1
1	0

Table 3

(d) Draw the switching circuit of each of the following function and replace them by Simpler one's :

(i) $F(x, y, z) = x \cdot y + [\, z \cdot (x' + y')\,]$

(ii) $F(x, y, z) = (x + y + z) \cdot (x' + y)$

Unit-II

2. (a) State and explain types of Binary relations.

(b) If I is the set of integers and the relation $x\,Ry \Rightarrow x - y$ is an even integer then prove that R is an equivalence relation where $x, y \in I$.

(c) Show that the function $f : R \rightarrow R$ defined by $f(x) = 3x + 5$ is one-one onto where R is the set of real numbers.

(d) If L be the set of all factors of 12 and let "1" be the divisibility relation on L. Show that $(L, 1)$ is a lattice.

Unit-III

3. (a) State and explain Group.

(b) Prove that the set fourth roots of unity (namely, $1, -1, i, -i$) form an abelian group with respect to multiplication.

(c) Prove that set of all integers I is a ring with composition of addition and multiplication.

(d) If G be the multiplicative group of the set $\{1, -1\}$ and G' be the additive group of residue class modulo 2 i.e. $G' = \{(0, 1), +2\}$ then show that there are isomorphic group.

Unit-IV

4. (a) State and explain Binary tree.

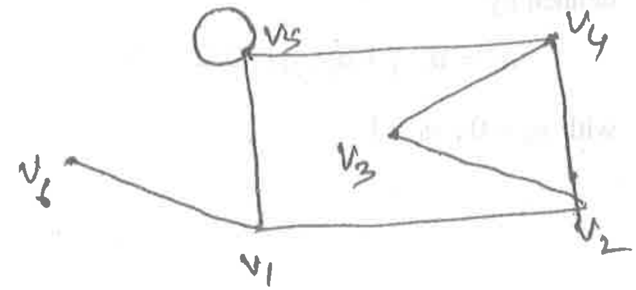
(b). (i) Prove that sum of the degree of all vertices in a Graph G is equal to twice the number of edges in G.

(ii) Prove that the vertices of odd degree in a graph is always even

(c) (i) Draw the graph whose incidence matrix M is given by :

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

(ii) Write the adjacency matrix of the graph shown in fig. below.



(d) Find the maximum and minimum possible height of a binary tree with 13 vertices and draw graph of the tree.

Unit-V

5. (a) Find the generating functions of the numeric function

$$a_r = 7 \cdot 3^r, r \geq 0$$

(b) How many positive inteters not exceeding 500 are divisible by 7 or 11?

(c) Solve the difference equation

$$a_{r+2} - 5a_{r+1} + 6a_r = 5^r$$

(d) Find the generating function for the sequence $\{ a_r \}$ defined by

$$a_r = a_{r-1} + a_{r-2}, r \geq 2$$

with $a_0 = 0$, $a_1 = 1$.